Grade 7 Mathematics, Quarter 4, Unit 4.1

Using Formulas to Find Area and Circumference

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Solve real-world problems using the formula for area of a circle.
- Solve real-world problems using the formula for circumference of a circle.
- Give ideas about the relationship between the area and circumference of a circle.
- Solve problems involving area of irregular shapes that are composed of triangles, quadrilaterals, and other polygons.

Mathematical practices to be integrated

Model with mathematics.

- Apply what they know about formulas for the area of shapes to solve real-life problems.
- Create irregular polygons and decompose them and find the area of the shapes used to make the polygons.

Use appropriate tools strategically.

- Use paper and pencil to calculate the area of shapes.
- Use tangrams to compose irregular polygons.

Attend to precision.

- Accurately calculate area and circumference using appropriate symbols.
- Justify the symbols and measurements used by relating to the formulas for finding area and circumference.

Essential questions

- What is the formula for area of circle?
- What is the formula for circumference of a circle?
- What real-world situation would require you to find the area of a shape?
- How do you find circumference of a circle, when given the radius?
- How do you find the area of a circle, when given the diameter?
- How do you find area of a polygon that is not a rectangle, square, or triangle?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	7.G

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.6 Solve real-world and mathematical problems involving area, ~~volume and surface area~~ of two- ~~and three-~~dimensional objects composed of triangles, quadrilaterals, polygons, ~~cubes, and right prisms~~.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 6, students found the area of polygons by decomposing polygons to make rectangles or triangles, and then finding the area of those shapes. They began formulating ideas about the formulas for finding area of given shapes.

Current Learning

In grade 7, students learn geometric concepts as additional clusters. These cannot be ignored, however they are unrelated to the major clusters of the grade level. Students solve real-life problems that require finding the area and circumference of circles using a formula—which has not been previously introduced—and by using formulas to find the area of triangles, polygons, and quadrilaterals. Students begin to recognize the relationship between area and circumference of circles.

Future Learning

In grade 8 and beyond, students will prove similarity, model using geometric figures, and make arguments about the relationships between shapes using area, volume, and angle measures of two- and three-dimensional shapes.

Additional Findings

According to the *PARCC Model Content Frameworks for Grade 7* (p. 32), teaching 7.G.6 provides students the opportunity for in-depth focus on previous workings with geometric measurement.

According to *Adding it Up* (p. 283), students find it difficult to decompose and then recompose shapes or see a large shape as a compilation of smaller different shapes; however, this understanding is fundamental to the idea of conservation.

Grade 7 Mathematics, Quarter 4, Unit 4.2

Using Proportional Relationships to Construct Similar Geometric Figures

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Compute area of a fractional pieces of units such as ratios of lengths, areas, and other measured quantities.
- Create scaled drawings of geometric figures.
- Compute actual lengths and areas from scaled drawings of geometric figures.
- Reproduce drawings of geometric figures at different scales.

Mathematical practices to be integrated

Make sense of mathematics and persevere in solving them.

- Analyze relationships between scaled drawings and actual lengths or areas.
- Use given information to plan for a solution to the problem.

Reason abstractly and quantitatively.

- Consider units involved and attend to the meaning of quantities to solve problems.

Construct viable arguments and critique the reasoning of others.

- Argue that lengths and areas of actual objects are accurate based on the scaled drawing and vice versa.

Essential questions

- How does a scaled drawing help you find the area of an actual object?
- How does a scaled drawing represent the actual length of a scaled object?
- How does the length of an actual object compare to the scaled drawing of that object?
- How do you adjust a drawing to reflect different scales?
- How do you find the area of a section of a larger object?
- How do you find unit rate with ratios of fractions?

Written Curriculum

Common Core State Standards for Mathematical Content

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.*

Geometry 7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative

relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

In grade 6, students used ratios and proportional reasoning to convert measurement units to multiply and divide quantities. Students solved problems involving unit rates of whole numbers and used the language of ratios and rates to understand the structure of rate tables.

Current Learning

In grade 7, students extend proportional reasoning to include all rational numbers. Students use this understanding to compute actual lengths of scaled drawings and reproduce drawings at different scales and use the proportional relationships between scaled drawings to find area.

Future Learning

In grade 8, students will reason about the similarity and congruence of two-dimensional scaled drawings and the relationships formed by intersecting lines.

Additional Findings

According to *Adding it Up* (p. 417), children need to develop all aspects of proportional reasoning. They can deepen their understanding of proportional relationships by using concrete materials to develop representations.

According to *Principles and Standards for School Mathematics* (p. 280), representation is key to the study of mathematics. Students deepen understanding by creating various representations of mathematical concepts such as charts, concrete models, and drawings.

Grade 7 Mathematics, Quarter 4, Unit 4.3

Solve Problems Involving Surface Area of Three-Dimensional Shapes

Overview

Number of instructional days: 8 (1 day = 45–60 minutes)

Content to be learned

- Describe the plane sections that come from slicing right rectangular prisms and right rectangular pyramids.
- Solve real-world problems involving surface area of cubes.
- Solve real-world problems involving surface area of right prisms.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use paper and pencil to calculate the surface area of three-dimensional shapes.

Attend to precision.

- Accurately calculate surface area of three-dimensional cubes and right prisms.
- Justify the symbols and measurements used.

Essential questions

- What plane sections result from slicing a right rectangular prism?
- What plane sections result from slicing a right rectangular pyramid?
- When would finding surface area of right prism occur in your everyday life?
- When would finding surface area of a cube occur in your everyday life?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.6 Solve real-world and mathematical problems involving area, ~~volume~~ and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students

give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 6, students found the area of polygons by decomposing polygons to make rectangles or triangles, and then found the area of those shapes. They began formulating ideas about the formulas for finding area of said shapes. Sixth grade students used nets to construct three-dimensional figures and used the nets to find surface area of these figures.

Current Learning

In grade 7, students learn geometric concepts as additional clusters. These cannot be ignored, however they are unrelated to the major clusters of the grade level. Students describe the two-dimensional figures that come from slicing right rectangular prisms, cubes, and right rectangular pyramids. They also find the surface area of those plane sections.

Future Learning

In grade 8 and beyond, students will prove similarity, model using geometric figures, and make arguments about the relationships between shapes by involving area, volume, and angle measures of two- and three-dimensional shapes. Students will describe three-dimensional figures by examining cross sections of two-dimensional figures.

Additional Findings

According to *Principles and Standards for School Mathematics* (p. 97), students must be allowed to physically manipulate geometric shapes by decomposing and recomposing. They then take this knowledge and describe three-dimensional shapes.

According to *Adding it Up* (p. 283), students find it difficult to decompose and then recompose shapes or see a large shape as a compilation of smaller different shapes; however, this understanding is fundamental to the idea of conservation.

Grade 7 Mathematics, Quarter 4, Unit 4.4

Solve Problems Involving Volume

Overview

Number of instructional days: 7 (1 day = 45–60 minutes)

Content to be learned

- Solve problems involving volume of cubes by using formulas.
- Solve problems involving volume of right prisms by using formulas.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use paper and pencil to calculate the volume of three-dimensional right prisms and cubes.
- Use manipulatives to fill and justify $V=Bh$.

Attend to precision.

- Accurately calculate the volume of three-dimensional shapes.
- Be careful to recognize volume as cubic units instead of square units.

Recognize that B is not base, but the area of the base.

Essential questions

- What are the formulas for volume?
- Why do the two formulas result in the same answer?
- When would you use volume in a real-world situation?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**7.G**

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 6, students found the volume of right rectangular prisms with fractional edges by packing the shape with unit cubes and applied the formula $V=l$ x w x h and $V=Bh$.

Current Learning

In grade 7, students solve problems by finding the volume of cubes and right prisms.

Future Learning

In grade 8, students will complete their work with volume by solving problems involving cones, cylinders, and spheres.

Additional Findings

According to the *PARCC Progressions K–6 Geometry* document *(*p. 19), students must learn to apply strategies and formulas for finding area and volume to solutions of real-world and mathematical problems.

According to *Adding it Up* (p. 284)*,* children understand *V=Bh* because they typically organize volume in three-dimensional arrays, where the area of the base is stacked to the height.

Grade 7 Mathematics, Quarter 4, Unit 4.5

Draw Figures and Solve Problems Using Angle Measures

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Draw geometric shapes with given conditions.
- Write simple equations for unknown angles in a multistep problem by using facts about supplemental, complementary, vertical and adjacent angles.
- Solve simple equations for unknown angles in a multistep problem by using facts about supplemental, complementary, vertical and adjacent angles.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use rulers and protractors to draw geometric shapes.

Attend to precision.

- Solve equations involving missing angle measures accurately.
- Know and accurately use definitions of facts about supplementary, complementary, vertical, and adjacent angles.

Essential questions

- What tools can help you draw geometric figures when given angle measurements?
- How do you use an equation to find a missing angle measure?
- What is a supplementary angle?
- What is a complementary angle?
- What is a vertical angle?
- What is an adjacent angle?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**7.G**

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In fourth grade, students drew and identified acute, obtuse, and right angles. In fifth grade, students classified geometric shapes based on the angle measures and other attributes. Sixth grade students drew polygons in the coordinate plane when given the coordinates for the vertices and found lengths of the sides by joining points with the same first coordinate.

Current Learning

Seventh grade students use angle measures to draw geometric figures. They know the rules regarding supplementary, complementary, adjacent, and vertical angles to write simple equations to find missing angle measures.

Future Learning

In grade 8, students will extend their knowledge of shapes and angles to use lines and angles to create similar figures, and to describe and analyze two-dimensional figures.

Additional Findings

According to the *PARCC Content Frameworks for Mathematics Grade 7* document (p. 32), working with angle measures and drawing figures brings together work with geometric measurement in grades 3–6.

According to *Principles and Standards for School Mathematics* (p. 233), middle grade geometry programs involving investigation lead to development of mathematical reasoning, validating conjectures, and classifying and defining geometric shapes. Many of the standards covered in middle grades are closely related to the study of geometry.